The Statistical Analysis of Strategic Behavior

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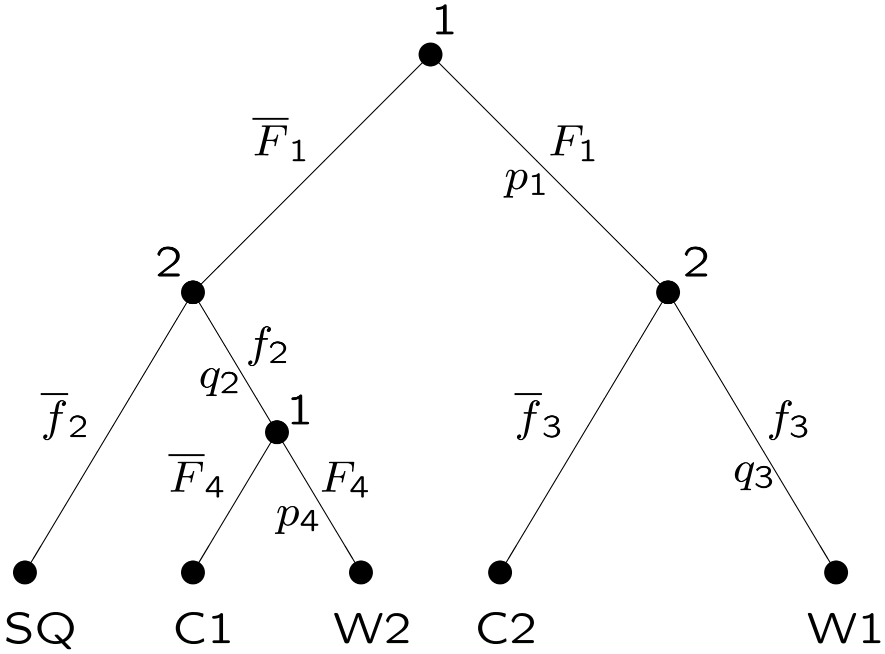
IR (and Other) Motivation

- IR theory = strategic interaction
  - Realism, neorealism, neoliberalism
  - Balance of power, arms races, war, deterrence, alliances...
- Large subset: finite choices → finite outcomes
  - Crises: SQ, Acq, Nego, Cap, War
  - Deterrence: Success, failure
  - Alliances: Balance, bandwagon, neutral
- Other areas
  - Vote Democrat, Republican, abstain
  - Bill passes or doesn't pass

How Can We Statistically Analyze Conflict?

- Exploratory data analysis?
- Testing "loosely-specified" theories
  - Regress "primitives" on outcome variable?
  - Regress some function of primitives on outcome variable?
- Testing formal theories
  - Assume Nash behavior and compare predicted vs actual outcomes?
  - Derive comparative statics and hope some are monotonic in the variables?
  - Derive a statistical version of the theoretical model?
- If structure matters, then first two may be problematic.

A Simple Crisis Interaction Model



$$
\begin{aligned}
U_i(SQ) &= D_{ij} \\
U_i(Ci) &= -A_i \\
U_i(Cj) &= A_j \\
U_i(W) &= P_i A_j + (1 - P_i)(-A_i - M_i) \\
P_i &= \frac{M_i}{M_i + M_j}
\end{aligned}
$$

- How do we test this model?
  - Assume Nash and compare predicted vs actual outcomes?
  - Derive comparative statics and hope some are monotonic?
  - Derive strategic statistical model.

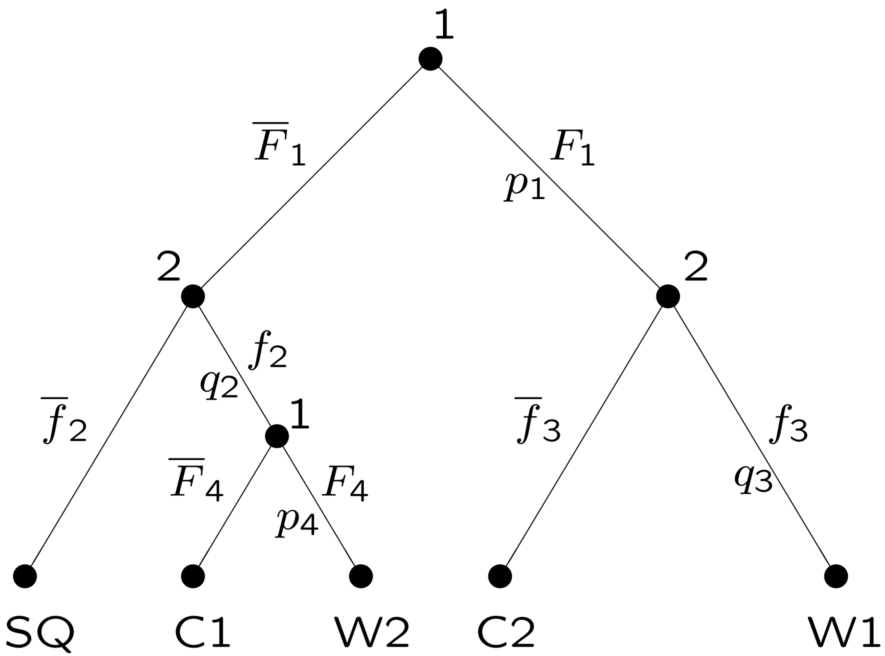
A Statistical Model Based on Agent Error

- McKelvey & Palfrey Logit Quantal Response Equilibrium (LQRE)
  - Utility maximizers: $U^*(a_j) > U^*(a_k)\ \forall k \neq j$.
  - Agent error: $U^*(a_j) = U(a_j) + \alpha_j$
  - Logit: α_j i.i.d. Type 1 Extreme Value
  - Results in strictly-positive equilibrium probabilities p_1, q_2, q_3, p_4
  - Outcome probabilities can be used in log-likelihood
  - Extent of agent "error" captured in variance parameter:
    $\lambda = 0 \rightarrow$ complete uncertainty
    $\lambda = \infty \rightarrow$ no error $\rightarrow$ here, subgame perfect equilibrium

LQRE Intuition

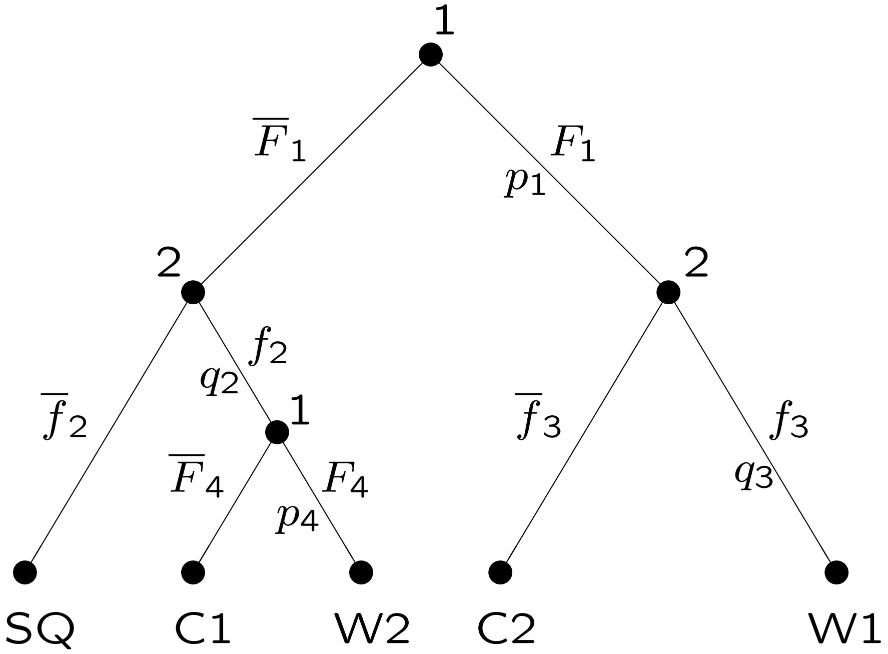
- People make mistakes, or intended actions not implemented as planned
- When engaged in competitive behavior, each player takes this into account.
- Choice probability at each information set is simply a logit probability.
- However, utility for an action leading to a non-terminal node is specified as expected utility.

LQRE Outcome Probabilities in the Crisis Interaction Model



$$
\begin{aligned}
p_{sq} &= (1-p_1)(1-q_2) \\
p_{c1} &= (1-p_1)q_2(1-p_4) \\
p_{w2} &= (1-p_1)q_2p_4 \\
p_{c2} &= p_1(1-q_3) \\
p_{w1} &= p_1q_3
\end{aligned}
$$

LQRE Choice Probabilities



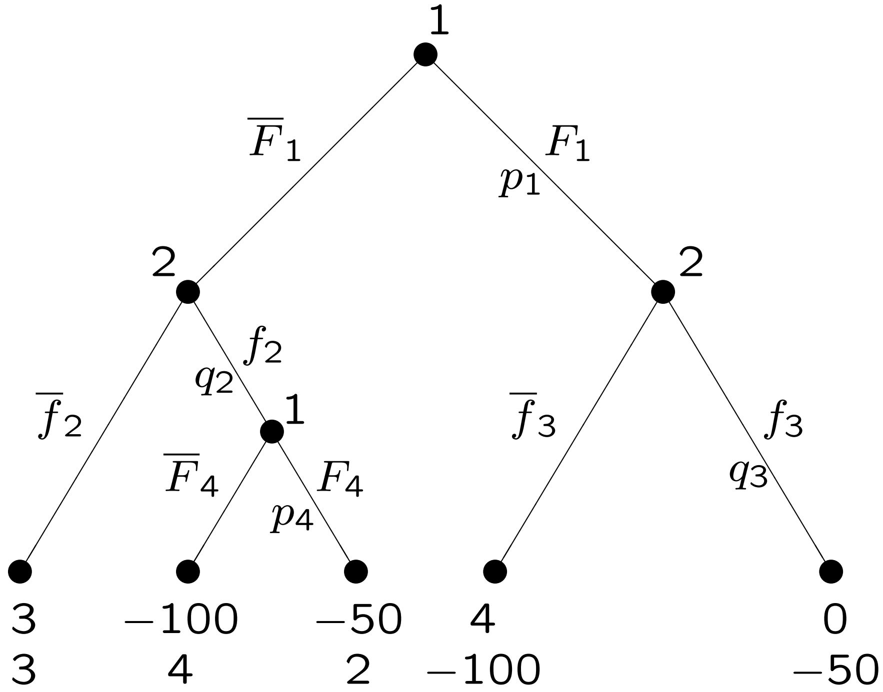
$$p_4 = \frac{e^{\lambda U_1(W2)}}{e^{\lambda U_1(W2)} + e^{\lambda U_1(C1)}}$$

$$q_3 = \frac{e^{\lambda U_2(W1)}}{e^{\lambda U_2(W1)} + e^{\lambda U_2(C2)}}$$

$$q_2 = \frac{e^{\lambda[p_4 U_2(W2)+(1-p_4)U_2(C1)]}}{e^{\lambda[p_4 U_2(W2)+(1-p_4)U_2(C1)]} + e^{\lambda U_2(SQ)}}$$

$$p_1 = \frac{e^{\lambda[q_3 U_1(W1)+(1-q_3)U_1(C2)]}}{e^{\lambda[q_3 U_1(W1)+(1-q_3)U_1(C2)]} + e^{\lambda\{q_2[p_4 U_1(W2)+(1-p_4)U_1(C1)]+(1-q_2)U_1(SQ)\}}}$$

Slightly More Concrete Equilibrium Examples



Case 1: No Uncertainty.
SPE is $\{(\bar{F}_1, F_4), (\bar{f}_2, f_3)\}$,
leading to SQ outcome.

Case 2: Complete Uncertainty.
$p_1 = q_2 = q_3 = p_4 = \frac{1}{2}$.

Case 3: Small amount of uncertainty: $\lambda = 1$

$p_1 = .999$
$q_2 = .269$
$q_3 = .999$
$p_4 = .999$

$\text{Pr}[SQ] = .00$
$\text{Pr}[War_1] = .99$

Extend to Nonexperimental Data

- Parameters

$$
\begin{aligned}
U_i(SQ) &= D_{ij} \\
U_i(Ci) &= -A_i \\
U_i(Cj) &= A_j \\
U_i(W) &= P_i A_j + (1 - P_i)(-A_i - M_i) \\
P_i &= \frac{M_i}{M_i + M_j}
\end{aligned}
$$

$$\beta_{m1} M_1,\ \beta_{m2} M_2,\ \beta_{a1} A_1,\ \beta_{a2} A_2,\ \beta_d D_{12}$$

- Likelihood function

$$L = \prod_{i=1}^{n} p_{sq}^{y_{i,sq}} p_{c1}^{y_{i,c1}} p_{w2}^{y_{i,w2}} p_{c2}^{y_{i,c2}} p_{w1}^{y_{i,w1}}$$

Monte Carlo Analysis

- All β's equal to 1, except $\beta_d = 20$
- Statistical strategic model recovered parameters

Logit Astrology

Naive: War = Military + Assets + Democracy

Parameters	Estimates	Std.Err.
$Constant$	.76	.293
$Military_1$	−.03	.003
$Military_2$	−.03	.003
$Assets_1$	.02	.003
$Assets_2$	.02	.003
$Democracy_{12}$	−1.15	.206
Mean $\ln L = -.5051$		N=1000

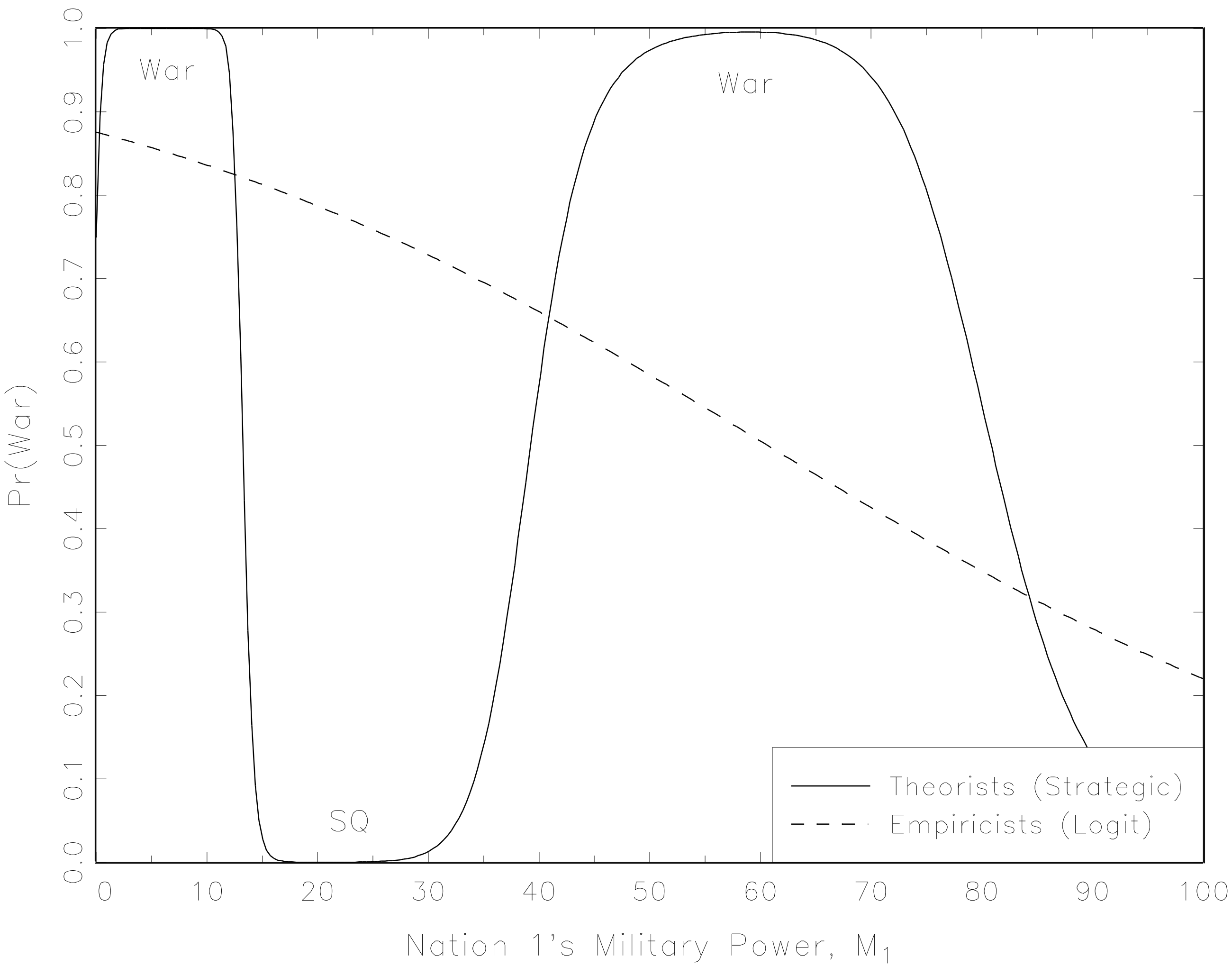
Balance of Power: War = MilCon + Democracy

Parameters	Estimates	Std.Err.
$Constant$	-1.45	.132
$MilCon$	3.20	.273
$Democracy_{12}$	-1.08	.189
Mean $\ln L = -.5808$		N=1000

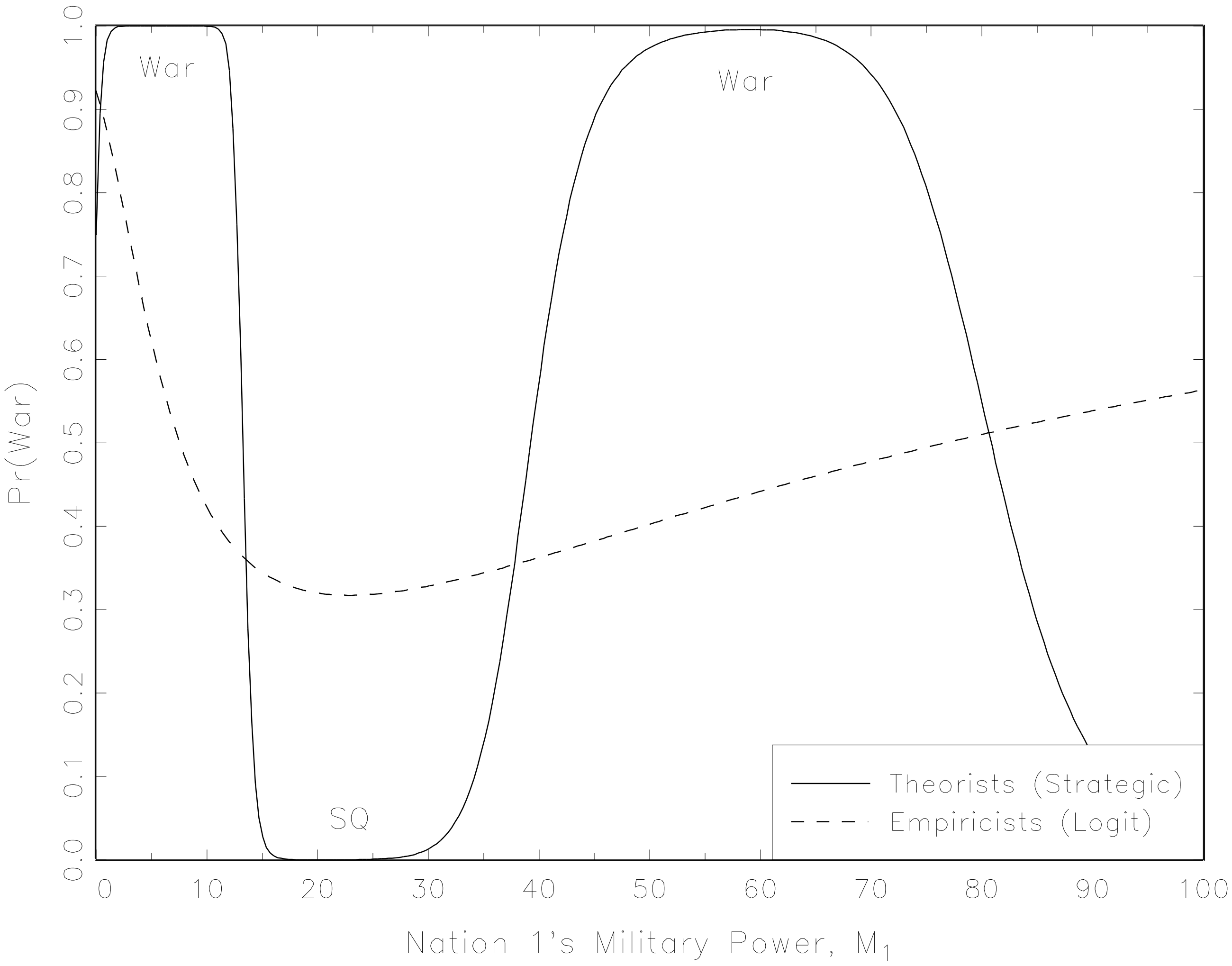
Expected Utility: War = $U_1(W)U_2(W)$ + Democracy

Parameters	Estimates	Std.Err.
$Constant$	$-.69$	.102
$U_1(W)U_2(W)$	$-.22$	.015
$Democracy_{12}$	-1.91	.301
Mean $\ln L = -.3516$		N=1000

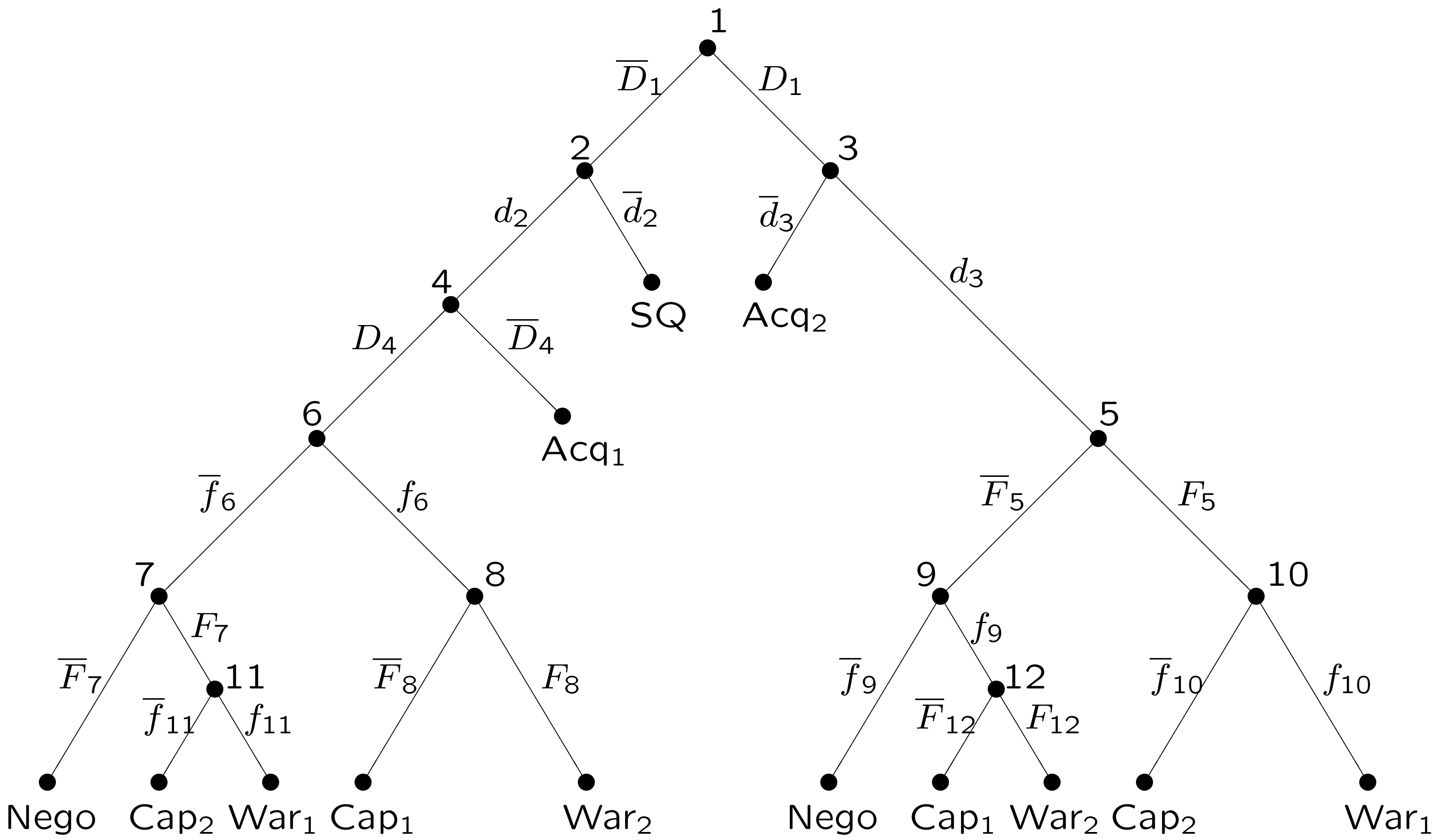
Military, Assets, Democracy



Joint Expected Utility of War



Reanalyzing the International Interaction Game



Comparing Observed Outcomes to Subgame Perfect Predictions

- **Problems with original BdM&L analysis**
  - Unavailable cost data $\rightarrow$ nonassessable equilibrium conditions
  - Tests based on aggregated outcomes

Bounds on performance

Sets of Possible Predicted Outcomes

Actual Outcome	$\emptyset$	SQ	Nego	Acq$_2$ Nego	Acq$_1$ Acq$_2$ Nego	Acq$_1$ Acq$_2$ Nego War$_1$	SQ Acq$_1$ Acq$_2$ War$_1$
SQ	21	$_1\mathbf{1}$	3	57	3	115	$_0\mathbf{38}$
Acq$_1$	0	0	0	1	0	$_0\mathbf{6}$	$_0\mathbf{1}$
Acq$_2$	4	1	0	$_0\mathbf{21}$	0	$_0\mathbf{62}$	$_0\mathbf{13}$
Nego	2	0	$_3\mathbf{3}$	$_0\mathbf{9}$	0	$_0\mathbf{11}$	10
Cap$_1$	0	0	1	5	0	18	2
Cap$_2$	6	0	1	17	0	74	12
War	20	1	5	36	0	$_0\mathbf{111}$	$_0\mathbf{16}$

$1\% \leq CP \leq 41\%$

A Statistical Model of the International Interaction Game

$$
\begin{aligned}
U_i(SQ) &= \beta_{i1} U_i(SQ') \\
U_i(Acq_j) &= \beta_{i2} U_i(\Delta_i) \\
U_i(Acq_i) &= \beta_{i3} U_i(\Delta_j) \\
U_i(Nego) &= \beta_{i4} P_i U_i(\Delta_i) + \beta_{i5}(1 - P_i) U_i(\Delta_j) \\
U_i(Cap_j) &= \beta_{i2} U_i(\Delta_i) + \beta_{i6} \phi_i P_i \\
U_i(Cap_i) &= \beta_{i3} U_i(\Delta_j) + \gamma_i (1 - P_i) \\
U_i(War_i) &= \beta_{i4} P_i U_i(\Delta_i) + \beta_{i5}(1 - P_i) U_i(\Delta_j) + \beta_{i6} \phi_i P_i + \alpha_i (1 - P_i) \\
U_i(War_j) &= \beta_{i4} P_i U_i(\Delta_i) + \beta_{i5}(1 - P_i) U_i(\Delta_j) + \beta_{i6} \phi_i P_i + \tau_i (1 - P_i)
\end{aligned}
$$

Models Estimated

- **Null Model**:
  $\beta_{ij} = \gamma_i = \alpha_i = \tau_i = 0$

- **BdM&L Bounded Rationality Model**:
  $\beta_{i1} = \beta_{i2} = \beta_{i3} = \beta_{i4} = \beta_{i5} = 1$ and $\beta_{i6} = -1$
  γ_i, α_i, τ_i estimated subject to: $\gamma_i, \alpha_i, \tau_i \leq 0$ and $\tau_i \leq \alpha_i$.

- **Unrestricted Model**:
  All parameters estimated.
  β_{ij} estimated subject to: $\beta_{i1}, \beta_{i2}, \beta_{i3}, \beta_{i4}, \beta_{i5} \geq 0$ and $\beta_{i6} \leq 0$.
  γ_i, α_i, τ_i estimated subject to: $\gamma_i, \alpha_i, \tau_i \leq 0$ and $\tau_i \leq \alpha_i$.

Term		Model 0 Null	Model 1 BdM&L-BR		Model 2 Unrestricted	
			Natn 1	Natn 2	Natn 1	Natn 2
$U(SQ')$	β_{i1}	0	1	1	0	$.87^*$
					.	(.24)
$U(\Delta_i)$	β_{i2}	0	1	1	.29	0
					(.18)	.
$U(\Delta_j)$	β_{i3}	0	1	1	2.55^*	$.91^*$
					(.38)	(.15)
$P_iU(\Delta_i)$	β_{i4}	0	1	1	0	0
					.	.
$(1-P_i)U(\Delta_j)$	β_{i5}	0	1	1	2.68^*	1.07^*
					(.66)	(.22)
ϕ_iP_i	β_{i6}	0	-1	-1	0	0
					.	.
$(1-P_i)$	γ_i	0	-1.50^*	0	$-.42$	0
			(.53)	.	(.54)	.
	α_i	0	0	0	0	0
			.	.	.	.
	τ_i	0	0	$-.39$	0	0
			.	(.26)	.	.
	$\ln L =$	-1292.7	-1286.5		-1207.9	
	$2(\ln L - \ln L^0) =$				169.5	
	$df =$				14	
	$p =$				0.000	
	$CP =$	14%†, 34%‡	23%		29%	

$N = 707$, †Acq$_2$ predicted, ‡SQ predicted, $^*p < .01$

Summary of Thoughts at This Point

- Seem to be problems with traditional estimation methods applied to strategic data.

- McKelvey & Palfrey provide a statistical equilibrium concept (QRE) that can be used in a regression context.

- Used LQRE to show problems with traditional methods, which are sometimes wildly off.

- Applied to well-known international interaction game.

- Q: Does connecting the theory and statistical model mean our results are "correct"?

Outstanding Issues

- Formal characterization of the misspecification
- Econometric perspective vs theoretical perspective
- How can we tell if strategic model is better than plain old logit (when we're not generating the data)?
- Multiple equilibria
- Others...

Later Today: LQRE Exercise using Mathematica

```
f[x_,y_]:=x^2+y^2

x=2

Plot[f[x,y],{y,.01,20}]
```