EITM Spatial Modeling Homework: Most of these should be easy, but some of them are relatively hard. Even easy problems can help you get the concepts clear, which is why they are included as well as the hard ones. Please complete this assignment by Saturday morning at 10:00 am, at which time we will meet to discuss it.

1. Consider an electoral game with three alternatives, A, B, and C, and assume that the electorate has the following preferences: APB, BPC, and CPA (where P = "is preferred to"). Assume that there are two purely office-seeking political parties, and they can each choose a pure strategy of either A, B, or C.

a. Write down the normal form of this game.

b. Is there an equilibrium in pure strategies in this game?

c. A mixed strategy by a party involves choosing a probability distribution over the three alternatives and choosing a strategy according to this distribution. That is, the party chooses an option according to the probability distribution (e.g., if the probability over option A is 1/4, then that option is chosen 1/4 of the time.) Is there a mixed strategy equilibrium in this game? (Optional: Is there a general theorem that can be appealed to here?)

d. What is the mixed strategy equilibrium? (Hint: Use the symmetry in this game.)

e. What is the payoff to each party given this mixed strategy equilibrium?

f. What do you think about mixed strategies as a solution to this game?

2. Consider the two-dimensional (X,Y) policy space with four voters at the following points:

(0,0), (1,1) (0,1) (1,0)

a. Is there an equilibrium solution (Condorcet winner) for these four voters? If there is such a solution, where is it?

b. Suppose another voter enters the electorate at (1/2, 2/5). Is there an equilibrium solution now? If there is such a solution, where is it?

3. Consider parties with multiple objectives or with different objectives.

a. Write down the maximand for a party in a two-party plurality electoral system that cares about maximizing an objective function that includes the benefits ("rents") from attaining office, the benefits from implementing a policy, and the benefits of simply expressing a policy.

b. Assume that there are three parties and one dimension of policy conflict. Suppose preferences are uniformly distribution along [0,1]. Suppose that voters vote for the party closest to them. Suppose that one party, which we call *A,* is merely expressive and wants to express a policy at point *A* along [0,1]. Suppose the other two parties, *B* and *C*, are merely office-seeking. Also assume that the closest any party can get to another is ε away, where ε is an arbitrarily small but positive and fixed number.

i. What happens if *A=0*? Is there an equilibrium? If there is, what is it and who wins?

ii. What happens if *A=0* and parties *B* and *C* have an equal positive "valence" advantage so that every voter has some extra utility from supporting *B* or *C* (but not *A*). (One way to think about this is that the valence advantage comes from *B* and *C* having convinced voters that a vote for *A* is "wasted.") Is there an equilibrium? If so, how does it change as the valence advantage goes from zero to "infinity?"

iii. What happens if *A = 1/2*? Is there an equilibrium? If so, what is it and who wins? What would happen with a valence advantage for *B* and *C* as described in (ii)?

iv. What happens if *A = 3/4*? Is there an equilibrium. If so, what is it and who wins? What would happen with a valence advantage for *B* and *C* as described in (ii)?

v. What happens if *A = 2/3*? Is there an equilibrium? If so, what is it and who wins?

vi. What is the general solution to this problem for arbitrary value of A?

vii. Assume that third parties in America are merely expressive. Does it matter where those parties locate themselves? (Compare Perot's Reform party, which was essentially near the center of the voter distribution, with Nader's Green party or Buchanan's Reform party, which were at the end of the distribution. Which party poses the greater danger to the established parties, or does it depend?)

4. Assume that the i^{th} party's utility for policy X is given by the following utility function:

$$U_i(X) = -(X - Z_i)^2$$

where Z_i is the i^{th} party's bliss point. Assume there are two parties, party zero with $Z_0 = 0$ and party one with $Z_1 = 1$. The parties can commit to a platform that will be implemented should the party win the election. The parties are uncertain about the median voter's preferred policy, X_m, and assign a uniform probability distribution between ($\frac{1}{2} - a$) and ($\frac{1}{2} + a$) to X_m, where a is in the open interval (0,1). The parties are exclusively policy motivated. Let X_0 and X_1 be the policies proposed by parties 0 and 1 respectively.

a. Write down the maximand for each party in terms of their utility functions and expectations about winning, depending upon the policies that they and the other party propose.

b. Show that the parties will never choose their bliss points and that they will never converge completely.

c. Solve for the equilibrium policy, given that it is symmetric, that is, $X_0 = 1 - X_1$. Discuss how the equilibrium depends on the level of uncertainty, described by *a*.

d. Show that the equilibrium must be symmetric.

5. We typically define the positions of voters and the positions of parties to be on the same scale so that we can compare them, but we do not concern ourselves very much with the zero point of these scales. But issues are typically "polarized" so that there are two "sides" to them. This suggests that it might make sense to consider issue scales with a defined zero point that would be the "middle" position. Suppose that there is a well-defined zero point for issues. Suppose that person i's position on the t^{th} issue is X_{it} and the j^{th} party's position on the t^{th} issue is β_{tj}.

a. Would the location of these zero points make any difference for the standard squared Euclidian distance utility function in which person i's utility for the j^{th} party is the following:

$$U_{ij} = - \Sigma_t (X_{it} - \beta_{tj})^2.$$

If the location does make a difference, what kind of difference would it make? What is the optimal choice of β_{tj} for a party to appeal to person i?

b. Would the location of these zero points make any difference in the following expression proposed by Rabinowitz and Macdonald for people's preferences in a spatial model:

$$U_{ij} = \Sigma_t X_{it}\beta_{tj}.$$

If they do make a difference, what kind of difference would they make? What is the behavior of this model? What is the optimal choice of β_{tj} for a party to appeal to person i? Rabinowitz and Macdonald propose that parties must locate within a "region of acceptability" that limits the size of β_{tj}. Why do they do this? What would happen if there were no such region?

c. Compare what Rabinowitz and Macdonald propose with the "traits" model in "Traits versus Issues" by Brady. What are the similarities and differences? (See especially pages 113 and 114). Could the set-up in Brady be used to test the Rabinowitz and Macdonald theory versus the squared Euclidian distance model? (Consider also Iversen, 1994, especially page 51.)