

PROBLEM 8.1 PdT

- Three types of voters, $J \in \{1, 2, 3\}$, each of size 1
- VOTER PREFS:

$$W^J = u(c^J) + H(g) = u(1 - \tau + f^J) + H(g)$$

↑ private cons. ↑ pub. good ↑ income: exog. & equal ↑ Group-specific Transfer
 lump-sum tax ⇒ no distortion

$u(\cdot)$ & $H(\cdot)$ both increasing & concave. ⇒ $u', H' > 0$; $u'', H'' < 0$

- POLICY: $q = [\tau, g, r, \{f^J\}] \geq 0$
- ↑ tax ↑ pub-good ↑ rents ↑ transfers ← all weakly positive

⇒ BUDGET CONSTRAINT: $3\tau = \sum_J f^J + g + r$

↑ revenue

- OFFICE-SEEKING, RENT-SEEKING PARTIES:

$$E(V^P) = P_P \cdot (R + \gamma r)$$

↑ prob. p wins ↑ exog. "ego" rents ↑ endog. "corruption" rents, inefficiently gleaned from budget (γ)

- CREDIBLE COMMITMENT TO PLATFORMS IN "PR" = single District

⇒ $P_P = \text{Prob} \left[\frac{1}{3} \sum_J \pi_{PJ} \geq \frac{1}{2} \right]$

↑ vote-share of party P in group J

- PdT-style PROBABILISTIC VOTING

⇒ $i \in J$ votes P iff $W^J(q_P) > W^J(q_{P'}) + (\delta + \sigma^{iJ})$

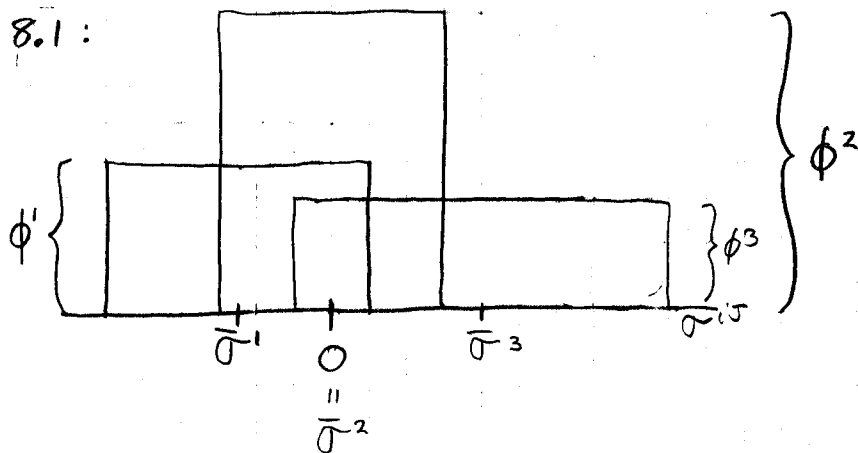
- where δ common "ideology" shock, $\delta \sim U[-\frac{1}{2\phi}, \frac{1}{2\phi}]$
- σ^{iJ} = individual, idiosyncratic shock, $\sigma^{iJ} \sim U[-\frac{1}{2\phi^J} + \bar{\sigma}^J, \frac{1}{2\phi^J} + \bar{\sigma}^J]$

Group J's Bias (J's density) zero-mean pro-P' shock Global Uncertainty

• ASSUMPTIONS REGARDING GROUP CHARACTERISTICS

- $\bar{\sigma}^1 < \bar{\sigma}^2 = 0 < \bar{\sigma}^3 \Rightarrow$ ($J=2$) on average neutral
 ($J=1$) pro-P; ($J=3$) pro-P'
- $\phi^2 > \phi^1, \phi^3 \Rightarrow$ Middle grp higher density (\approx "clout" in DL)
- $\bar{\sigma}^1 \phi^1 + \bar{\sigma}^3 \phi^3 = 0 \Rightarrow \sum_J \bar{\sigma}^J \phi^J = 0$ } An sneakily useful assumption whose meaning is that individualistic shocks cancel out!

\Rightarrow FIG 8.1:



• consider a deviation by A from $q_A = q_B$

- $\downarrow r$ or $\uparrow q$ each would \Rightarrow rightward shift of each group's cutpoint by same amount, say σ' , \Rightarrow gain $\phi^J \sigma'$ voters in each
- $\uparrow r \Rightarrow$ lose votes in all groups by opposite of above.
- $\uparrow f^J \downarrow f^{J'} \Rightarrow$ shift J cutpoint right, gaining $\phi^J \sigma''$, and J' left, losing $\phi^{J'} \sigma''$

\Rightarrow VOTE-SHARE PARTY A IN GROUP J: $\pi_{A,J}$

$$\pi_{A,J} = \phi^J [W^J(q_A) - W^J(q_B) - f - \bar{\sigma}^J] + \frac{1}{2}$$

• Density \cdot [Net Advantage] + starting vote

- Net Advantage: ① Policy Advantage: $\{W^J(q_A) - W^J(q_B)\}$
 ② (Common) Ideology Shock Advantage
 ③ Mean Group-Advantage
- uncertainty part* \rightarrow

• What's P_A ?

(3)

$$\pi_{A,T} = \phi^T [W_A^T - W_B^T - f - \sigma^T] + \frac{1}{2}$$

$$P_A = \text{Prob} \left[\frac{1}{3} \sum_T \pi_{A,T} \geq \frac{1}{2} \right]$$

$$\Rightarrow \text{Prob} \left[\frac{1}{3} \sum_T \phi^T [W_A^T - W_B^T - f - \sigma^T] > 0 \right]$$

the $\frac{1}{2}$'s cancel

$$\Rightarrow \text{Prob} \left[\frac{1}{3} \sum_T \phi^T [W_A^T - W_B^T - \sigma^T] > \frac{1}{3} \sum_T \phi^T f \right]$$

bringing f out

$$\Rightarrow \text{Prob} \left[\frac{\sum_T \phi^T [W_A^T - W_B^T - \sigma^T]}{\sum_T \phi^T} > f \right]$$

\Rightarrow (sneaky Assumption):

$$P_A = \text{Prob} \left[\frac{1}{3\phi} \sum_T \phi^T \{W_A^T - W_B^T\} > f \right]$$

• For any $X \sim U[a, b]$, $\text{Prob } X < C = \frac{C-a}{b-a}$

$$\Rightarrow \text{Prob}(f < C) = \frac{C - (-\frac{1}{24})}{\frac{1}{24} - (-\frac{1}{24})} = \frac{C + \frac{1}{24}}{\frac{1}{24} + \frac{1}{24}}$$

$$= \frac{C + \frac{1}{24}}{\frac{1}{12}} = \psi C + \frac{1}{2}$$

$$\Rightarrow P_A = \frac{\psi}{3\phi} \sum_T \phi^T \{W_A^T - W_B^T\} + \frac{1}{2}$$

So, question (a):

$$\text{Max}_{\{z, \{f^T\}, g, r\}} \sum_T W^T(q) \quad \text{s.t.} \quad q \geq 0 \text{ \& } 3z = \sum_T f^T + g + r$$

• n.b., income equal
 \Rightarrow no redist.

• n.b., $r =$ pure loss
 \Rightarrow no rents

\Rightarrow find optimal $\{z, g\}$

$$\Rightarrow \text{Max}_{\{z, g\}} \sum_T W^T(q) \quad \text{s.t.} \quad 3z = g \text{ or } z = \frac{1}{3}g$$

$$\Rightarrow \text{Max}_{\{z, g\}} \sum_T [U(c^T) + H(g)] \quad \text{s.t.} \quad z = \frac{1}{3}g$$

$$\Rightarrow \text{Max}_g \sum_T U(1 - \frac{1}{3}g) + H(g)$$

$$\Rightarrow \frac{1}{3} U'(1 - \frac{1}{3}g) = H_g(g)$$

\Rightarrow not very interesting; size of gov't $\Rightarrow f$ (rel. marg. util. priv/pub)

⑥ Now Parties Maximizing $E(V^P) = p_p \cdot (R + \delta r)$ ④

$$\Rightarrow \text{Max}_{\{z_A, \{f_A^J\}, g_A, r_A\}} E(V^P) \text{ s.t. } 3z_A = \sum_J f_A^J + g_A + r_A$$

$$= \text{Max}_{g_A} p_A \cdot (R + \delta r) = \left[\frac{1}{2} + \frac{\psi}{3\phi} \sum_J \phi^J \{W_A^J - W_B^J\} \right] (R + \delta r) \text{ s.t. } \dots$$

$$= \text{Max}_{g_A} \left[\frac{1}{2} + \frac{\psi}{3\phi} \sum_J \phi^J \{u(1 - z_A + f_A^J) + H(g_A) - W_B^J\} \right] (R + \delta r)$$

$$\text{s.t. } 3z_A = \sum_J f_A^J + g_A + r_A$$

So, \mathcal{L}

$$\text{Max}_{g} \left[\frac{1}{2} + \frac{\psi}{3\phi} \sum_J \phi^J \{u(1 - z_A + f_A^J) + H(g_A) - W_B^J\} \right] (R + \delta r) - \lambda (\sum_J f_A^J + g_A + r - 3z_A)$$

$$\frac{\partial \mathcal{L}}{\partial f^J} = (R + \delta r) \cdot \frac{\psi}{3\phi} \cdot \phi^J \cdot u'(c^J) = \lambda \quad \forall J$$

$$\Rightarrow \phi^J u'(c^J) = \lambda \cdot \frac{3\phi}{\psi} \cdot (R + \delta r)^{-1} \quad \forall J$$

① $\phi^1 u'(c^1) = \phi^2 u'(c^2) = \phi^3 u'(c^3) \Rightarrow f_A^J$ increasing in DL cbut

② All $f_A^J \uparrow$ proportionately in (ψ, R, δ, r^*) but $r^* \dots$

$$\frac{\partial \mathcal{L}}{\partial r} = \delta p_A - \lambda = 0 \Rightarrow \delta p_A = \phi^J u'(c^J) \frac{\psi}{3\phi} \cdot (R + \delta r)$$

$$\Rightarrow R + \delta r = \frac{3\phi \delta p_A}{\phi^J u'(c^J) \psi}$$

$$\Rightarrow r^* = \frac{3\phi p_A}{\phi^J u'(c^J) \psi} - \frac{R}{\delta} \quad (\text{n.b., in eqbm } p_A = \frac{1}{2})$$

① $r^* \downarrow$ in ψ, R, δ

$$\frac{\partial \mathcal{L}}{\partial g} = \cancel{(R + \delta r)} \cdot \frac{\psi}{3\phi} \cdot \sum_J \phi^J \cdot H_g(g) = \lambda = \cancel{(R + \delta r)} \cdot \frac{\psi}{3\phi} \cdot \phi^J \cdot u'(c^J)$$

$$\textcircled{1} H_g(g) = \frac{\phi^J \cdot u'(c^J)}{3\phi}$$

$$\textcircled{2} \sum_J \phi^J H_g(g) = \lambda \cdot 3\phi \cdot \psi^{-1} \cdot (R + \delta r)^{-1}$$

$$\Rightarrow H_g(g) = \lambda \cdot \psi^{-1} \cdot (R + \delta r)^{-1}$$

$\Rightarrow g \uparrow$ proportionately in (ψ, R, δ, r^*)
but r^*

F.O.C.'s

$$0 = \frac{\partial \mathcal{L}}{\partial \pi^J} \Rightarrow (R + \delta r_A) \frac{\psi}{3\phi} \theta^J u'(C_A^J) = \lambda$$

$$0 = \frac{\partial \mathcal{L}}{\partial r} \Rightarrow \delta p_A = \lambda$$

$$0 = \frac{\partial \mathcal{L}}{\partial g} \Rightarrow (R + \delta r_A) \psi H_g(q_A) = \lambda$$

$$0 = \frac{\partial \mathcal{L}}{\partial \gamma} \Rightarrow \frac{1}{3} (R + \delta r_A) \cdot \frac{\psi}{3\phi} \cdot \sum_J \phi^J u'(C_A^J) = \lambda$$

$$0 = \frac{\partial \mathcal{L}}{\partial \gamma} \Rightarrow 3 c_A = \sum_J f_A^J + g_A + r_A$$

⇒ Need More Structure on $U(\cdot)$ & $H(\cdot)$ to get interesting...

NLS: $y_i = h(x_i, \beta) + \varepsilon_i$

$$\Rightarrow \frac{1}{2} \sum \varepsilon_i^2 = \frac{1}{2} \sum [y_i - h(x_i, \beta)]^2 \equiv SSE(\beta)$$

Min _{β} SSE $\Rightarrow \frac{\partial SSE(\beta)}{\partial \beta} = \sum_{i=1}^n [y_i - h(x_i, \beta)] \frac{\partial h(x_i, \beta)}{\partial \beta} = 0$

• The Normal Equation from LS; e.g., if $h(\cdot)$ linear

$$\Rightarrow \sum_{i=1}^n (y_i - X_i \beta) \cdot X_i = 0 \quad \text{i.e., } \sum_{i=1}^n \varepsilon_i X_i = 0$$

NLS: Use Taylor Series Approx. to $h(x, \beta)$

$$\Rightarrow h(x, \beta) \approx h(x, \beta^0) + \sum_{k=1}^K \frac{\partial h(x, \beta^0)}{\partial \beta_k^0} \cdot (\beta_k - \beta_k^0)$$

$$\Rightarrow h(x, \beta) \approx \left[h(x, \beta^0) - \sum_{k=1}^K \frac{\partial h(x, \beta^0)}{\partial \beta_k^0} \cdot \beta_k^0 \right] + \sum_{k=1}^K \beta_k \cdot \frac{\partial h(x, \beta^0)}{\partial \beta_k^0}$$

letting $X_k^0 = \frac{\partial h(x, \beta^0)}{\partial \beta_k^0} \Rightarrow h(x, \beta) \approx [h^0 - \sum X_k^0 \beta_k^0] + \sum X_k^0 \beta_k$

$$y = h(x, \beta) + \varepsilon \approx h^0 - X_i^0 \beta^0 + X_i^0 \beta + \varepsilon$$

⇒ Replace usual regressors, X , w/ pseudoregressors X^0 & β^0 & iterate until β^0 not updated noticeably
⇒ If pseudoregressors well-behaved, $\hat{\beta}$ all usual properties.