Optimal Delegation (Rogoff '85):

- Agent w/ param A can delegate perfectly to $\hat{A}$, but cannot force him to follow any policy but his preferred

$$\Rightarrow \max_{\hat{A}} E(V(A,\hat{A})) = \max_{\hat{A}} E\left[-\tfrac{1}{2}\left(\hat{A}y^T - \tfrac{\hat{A}}{1+\hat{A}}\varepsilon\right)^2 - \tfrac{A}{2}\left(\tfrac{1}{1+\hat{A}}\varepsilon - y^T\right)^2\right]$$

$$\frac{\partial E(V(\cdot))}{\partial \hat{A}} = E\left[-\left(\hat{A}y^T - \tfrac{\hat{A}}{1+\hat{A}}\varepsilon\right)\left(y^T - \tfrac{1}{(1+\hat{A})}\varepsilon + \tfrac{\hat{A}}{(1+\hat{A})^2}\right) + A\left(\tfrac{1}{1+\hat{A}}\varepsilon - y^T\right)\left(\tfrac{\varepsilon}{(1+\hat{A})^2}\right)\right]$$

$$= E\left[-\left(\hat{A}y^T - \tfrac{\hat{A}}{1+\hat{A}}\varepsilon\right)\left(y^T - \tfrac{\varepsilon}{(1+\hat{A})^2}\right) + A\left(\tfrac{\varepsilon}{1+\hat{A}} - y^T\right)\left(\tfrac{\varepsilon}{(1+\hat{A})^2}\right)\right]$$

$$= E\left[-\hat{A}y^{T^2} + \hat{A}y^T\varepsilon/(1+\hat{A})^2 + \tfrac{\hat{A}}{1+\hat{A}}\varepsilon y^T - \tfrac{\hat{A}}{(1+\hat{A})^3}\varepsilon^2 + \tfrac{A}{(1+\hat{A})^3}\varepsilon^2 - \tfrac{A}{(1+\hat{A})^2}\varepsilon y^T\right]$$

$$= -\hat{A}y^{T^2} + 0 + 0 - \tfrac{\hat{A}}{(1+\hat{A})^3}\sigma_\varepsilon^2 + \tfrac{A}{(1+\hat{A})^3}\sigma_\varepsilon^2 - 0$$

$$\Rightarrow \frac{\sigma_\varepsilon^2}{(1+\hat{A})^3}(A-\hat{A}) = \hat{A}y^{T^2}$$

$$\sigma_\varepsilon^2(A-\hat{A}) = (1+\hat{A})^3\hat{A}y^{T^2}$$

$$\sigma_\varepsilon^2 A = \sigma_\varepsilon^2\hat{A} + (\hat{A}+1)^3\hat{A}y^{T^2}$$

$$A = \hat{A} + \hat{A}(1+\hat{A})^3 y^{T^2}/\sigma_\varepsilon^2$$

$$\Rightarrow A > \hat{A}$$

Lohmann (92): Policymaker can fire banker at cost c

$$\Rightarrow \pi = \begin{cases} \pi_b^* & \text{for } z \text{ low} \\ \phi(z)\pi_g(z) + (1-\phi(z))\pi_b(z) \end{cases}$$

where z is random shock

Comparing $\pi = \pi_c = 0$ to $\pi = \pi_d = Ay^T - \frac{A}{1+A}\varepsilon$

$$V = -\tfrac{1}{2}\pi^2 - \tfrac{A}{2}(y-y^T)^2$$

$$= -\tfrac{A}{2}\cdot(\varepsilon - y^T)^2$$

$$= -\tfrac{A}{2}(\varepsilon^2 - 2\varepsilon y^T + y^{T2})$$

$$\Rightarrow E(V) = -\tfrac{A}{2}(\sigma_\varepsilon^2 + y^{T2})$$

$$V = -\tfrac{1}{2}\pi^2 - \tfrac{A}{2}(y-y^T)^2$$

$$= -\tfrac{1}{2}\left(Ay^T - \tfrac{A}{1+A}\varepsilon\right)^2 - \tfrac{A}{2}\left(\tfrac{1}{1+A}\varepsilon - y^T\right)^2$$

$$= -\tfrac{1}{2}\left((Ay^T)^2 - 2\cdot\tfrac{A^2}{1+A}\cdot y^T\cdot\varepsilon + \left(\tfrac{A}{1+A}\right)^2\varepsilon^2\right)$$

$$-\tfrac{A}{2}\left(\left(\tfrac{1}{1+A}\right)^2\varepsilon^2 - 2\left(\tfrac{1}{1+A}\right)\varepsilon y^T + y^{T2}\right)$$

$$\Rightarrow E(V) = -\tfrac{1}{2}\left[(Ay^T)^2 + \left(\tfrac{A}{1+A}\right)^2\sigma_\varepsilon^2\right]$$

$$-\tfrac{A}{2}\left[\left(\tfrac{1}{1+A}\right)^2\sigma_\varepsilon^2 + y^{T2}\right]$$

[mult both by $-\frac{2}{A}$]

Prefer Commit iff

$$\sigma_\varepsilon^2 + \cancel{y_T^2} < \tfrac{1}{A}\left[(Ay^T)^2 + \left(\tfrac{A}{1+A}\right)^2\sigma_\varepsilon^2\right] + \left[\left(\tfrac{1}{1+A}\right)^2\sigma_\varepsilon^2 + \cancel{y^{T2}}\right]$$

$$< Ay^{T2} + \tfrac{A}{(1+A)^2}\sigma_\varepsilon^2 + \tfrac{1}{(1+A)^2}\sigma_\varepsilon^2 \cancel{+ y^{T2}}$$

$$\sigma_\varepsilon^2 < Ay^{T2} + \tfrac{1}{1+A}\sigma_\varepsilon^2$$

$$\sigma_\varepsilon^2 - \tfrac{1}{1+A}\sigma_\varepsilon^2 < Ay^{T2}$$

$$\sigma_\varepsilon^2\left(\tfrac{A}{1+A}\right) < Ay^{T2}$$

PREFER COMMIT IFF $\Rightarrow$ $\boxed{\sigma_\varepsilon^2 < y^{T2}(1+A)}$